SUPPLEMENTARY MATERIALS

Modelling cervical motion as a ball joint

Relationship of head, neck, and trunk was modeled as a two rigid bodies connected by a ball joint and an articulated body that could be expressed in chain of SE(3) group matrix.

$$\begin{bmatrix} R_b(t) & T_b(t) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & T_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_n(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2 & T_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_h(t) & T_h(t) \\ 0 & 1 \end{bmatrix} \tag{1}$$

where, subscript b is body, n is neck, h is head, 1 means transformation from the body frame to the neck frame in initial configuration, and 2 means transformation from the neck frame to the head frame in initial configuration.

Each matrix is a frame that defines geometric location of the rigid bodies and the joint. Each frame contains an orientation/rotation matrix R (3×3 dimension) and a translation matrix to show translation from the parent frame T (3×1 dimension). The R_b, R_h matrices represent orientation. The other R matrices represent rotation. R_1, T_1, R_2, T_2 are constant matrices defined by initial configuration. Since human body is not a rigid body, translation part of the both side of the equation is not always the same. We only extracted rotation part of the equation and the rotation of the neck.

$$\mathrm{R_n}\ (t)=R_1^{-1}\times R_b^{-1}(t)\times R_h(t)\times R_2^{-1} \tag{2}$$

The rotation of the ball joint is divided into 3 axial plains for clinical usage: flexion/extension, lateral bending, and axial rotation. These clinical terminologies are not defined mathematically. We matched each cervical motion with the components of an Euler angle obtained from the rotation matrix.

Initial frame setting of the ball joint that represents the neck could be geometrically random. Axis were defined in initial frame of the neck when subject was in a natural position. X-axis was defined towards direction anterior of the body. Y-axis was for the left side, and Z-axis was for superior direction.

R_1, R_2 are constant matrix determined from the initial neck frame, when t=0

$$\begin{aligned} R_b(0)R_1&=R_n(0) \\ R_n(0)R_2&=R_h(0) \end{aligned} \tag{3}$$

Obtaining R_h and R_b is the same for optical motion capture or Kinect, but different for AHRS.

Any three points attached on rigid body can produce three orthonormal axes consisting each column of orientation matrix by the following procedure:

X axis=normalize (P2-P0)
Y axis=normalize (P1-P0)
Z axis=cross (X axis, Y axis)
X axis=cross (Y axis, Z axis)

The correspondence between markers and P0, P1, P2 are not fixed. For the body, we chose markers at 7th cervical vertebra (C7) as P0, CLAV as P1, and STRN as P2. For head, we chose RBHD as P0, RFHD as P1, and LBHD as P2. In this way, we obtained rotation of the body (R_b) and rotation of the head (R_h).